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DRIFT INSTABILITY OF A NONUNIFORM PLASMA IN A MAGNETIC FIELD

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1. As is well known, there are always drift flows of ions and electrons in a nonuniform plasma situated in a magnetic field. One may think, by analogy with the normal beam instability, that such flows must lead to a build-up of waves, whose phase velocity across the magnetic field is of the order of drift velocity. Waves of that sort, which may be designated as drift waves, really exist in a nonuniform plasma. Their study was initiated by the Tserkovnikov 's work [1]. Using as an example a plasma situated in the field of a direct current, it was shown that in the presence of a temperature gradient, drift-wave build-up, propagating across the magnetic field, may take place. A hydrodynamic approach leads to about the same stability condition [2]. An oblique drift wave was examined in the work by L. I. Rudakov and P. Z. Sagdeyev [3], which passed to the ion-acoustic wave when the angle between the wave vector and the magnetic field vector decreased. Such a wave damps very little at nearly transverse propagation, and in the presence of a temperature gradient or of a longitudinal current it may become a growing wave in time.

In contrast to the above-enumerated cases, the effect of finite Larmor radius of ions is taken into account in the present work.

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* Dreyfovaya neustoychivost' neodnorodnoy plazmy v magnitnom pole.

2. Let us consider a nonuniform plasma situated in a uniform magnetic field H, directed along the axis **z**. Limiting ourselves to a one-dimensional case, we shall consider that the equilibrium functions of electrons f_e and of ions f_i depend only on a single spatial coordinate **x**. If the functions f_j vary little over the length of the mean Larmor radius, we shall have in a system of coordinates where the mean electric field is absent :

$$f_j = f_{0j}(v_\perp^2, v_z, x) + \frac{v_y}{\Omega_j}\frac{\partial f_{0j}}{\partial x}, \qquad (1)$$

where $v_\perp^2 = v_x^2 + v_y^2$, $\Omega_j = e_j H / m_j c$, e_j is the charge, m_j is the mass of the particle of the kind **j**.

We shall consider here longitudinal-type waves, propagating across the density gradient. The electric field potential may be chosen for such waves in the form $\varphi' = \varphi \exp(-i\omega t + ik_z z + ik_y y)$, the solution of the kinetic equation for the perturbation of the distribution function f'_j of the **j** kind will be written in the form of an integral by trajectories (see for example [4]):

$$f'_j = i\varphi \frac{e_j}{m_j} \int_{-\infty}^{0} \mathbf{k} \frac{\partial f_j}{\partial \mathbf{v}(t)} \exp(-i\omega t + ik_z v_z t + ik_y y_j(t))\, dt, \qquad (2)$$

where $y_j(t) = \int_0^t v_{y_j}(t)\, dt = \frac{v_\perp}{\Omega_j}\cos(\Omega_j t - \psi) - \frac{v_\perp}{\Omega_j}\cos\psi$, ψ is the azimuthal angle in the velocity space. For perturbations with a wavelength much greater than the Debye radius, the dispersion equation may be obtained simply from the quasineutrality conditions $\sum_j \int e_j f'_j \, d\mathbf{v} = 0$. We shall assume that the distribution of electrons and ions along the transverse velocities is Maxwellian with temperatures $T_{\perp e}$ and $T_{\perp i}$ respectively. Since function (1) is not dependent on time, we may, after differentiation along V, take it out of the integral in time, and the dispersion equation will then transform to the form

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$$\sum_j \int \left\{ \frac{k_z}{m_j} \frac{\partial f_{0j}}{\partial v_z} + \frac{k_y}{m_j \Omega_j} \frac{\partial f_{0j}}{\partial x} - \frac{k_y v_{yj}(t)}{T_{\perp j}} f_{0j} - \right.$$

$$\left. - \frac{k_y v_{yj}(t) v_y}{m_j \Omega_j} \frac{\partial}{\partial x} \left(\frac{f_{0j}}{T_{\perp j}} \right) \right\} e^{-i\omega t + i k_z v_z t + i k_y y_j(t)} \, d\mathbf{v}\, dt =$$

$$= i \sum_j \int \left\{ \frac{F_j}{T_{\perp j}} + \sum_n (\omega - k_z v_z - n\Omega_j + i\nu)^{-1} \times \right.$$

$$\times \left[\frac{k_z}{m_j} \beta_{nj} \frac{\partial F_j}{\partial v_z} + \frac{k_y}{m_j \Omega_j} \frac{\partial}{\partial x} (\beta_{nj} E_j) - \right.$$

$$\left. \left. - \frac{\omega - k_z v_z}{T_{\perp j}} \beta_{nj} F_j - \frac{n^2 k_y}{m_j \Omega_j} \frac{\partial}{\partial x} \left(\beta_{nj} \frac{F_j}{b_j} \right) \right] \right\} dv_z = 0, \qquad (3)$$

where F_j is the distribution function from the longitudinal velocity, $\beta_{nj} = e^{-b_j} I_n(b_j)$; I_n is the Bessel function from an imaginary argument; b_j is a minor positive quantity introduced for a correct by-passing of poles.

3. Let us consider the oscillations of isotropic ($T_\perp = T_\parallel$) isothermic plasma with a Maxwellian distribution of electrons and ions along the longitudinal velocity v_z and a constant temperature ($dT/dx = 0$). Assuming the oscillation frequency is much lower than the cyclotron frequency of ions Ω_i, we may neglect all terms in the sum by n in (3), except the zero term. Limiting ourselves besides that to oscillations with a transverse wavelength, much greater than the mean Larmor radius of electrons, we shall postulate $b_e = 0$. Under these assumptions equation (3) is brought to the form

$$(z + \alpha)\, \beta Y(z) - p\,(z - \alpha)\, Y(pz) = 2 \qquad (4)$$

where $z = \frac{\omega}{k_z} \sqrt{\frac{m_i}{2T}}$; $b = k_y^2 \rho_i^2 = k_y^2 T / m_i \Omega_i^2$, $p = \sqrt{m_e/m_i} \ll 1$, $\alpha = -\sqrt{\frac{b}{2}} \times \frac{1}{k_z a}$, $\beta = e^{-b} I_0(b)$; I_0 is a Bessel function from an imaginary argument; $Y(z) = 2e^{-z^2} \int_0^z e^{t^2} dt - i\sqrt{\pi} e^{-z^2}$; a is the characteristic length over which the mean density of the plasma n_0: $a^{-1} = d \ln n_0/dx$ varies notably.

The investigation of equation (4) shows that at $\alpha < 0$, when the phase velocity of the wave is directed toward the drift of ions, the wave is distinctly damped, and besides its damping is exponentially small at $\alpha p \gg 1$. Such waves may be excited by longitudinal beams of electrons or ions. As to waves propagating toward the side electron drift ($\alpha > 0$), only the presence of density gradient appears to be sufficient for their growing in time.

Let us consider first of all the region $b \gtrsim 1$. For a sufficiently great α, i.e. for a small k_z, z will be sufficiently great, and we then shall have approximately $Y(z) \cong 1/z$. If on the other hand $p\alpha \ll 1$, pz will also be substantially smaller than the unity. At the same time, $Y(pz) \cong i\sqrt{\pi}$ $k_y^2 T_{\perp i}/m_i\Omega_i^2$, and from (4) we obtain :

$$\frac{\mathrm{Im}\,\omega}{k_y v_0} \cong \frac{\beta\sqrt{2\pi b}\,p}{k_z a}\,\frac{1-\beta}{(2-\beta)^2} \ll \frac{\nu p}{k_y v_0} \cong \frac{\beta}{2-\beta}, \qquad (T_e = T_i) \tag{5}$$

where $v_0 = T/am_i\Omega_i$ is the drift velocity.

As we may see, the increment of small perturbation growth $\mathrm{Im}\,\omega$ rises as $k_z a$ decreases. This rise lasts until the quantity $p\alpha$ becomes of the order of the unity, and when the expansion may no longer be applied to $Y(pz)$. The results of numerical computation of the increment near the maximum for the three values of b ($b = 0.02$, $b = 1$, $b = 10$) are plotted in Fig.1.

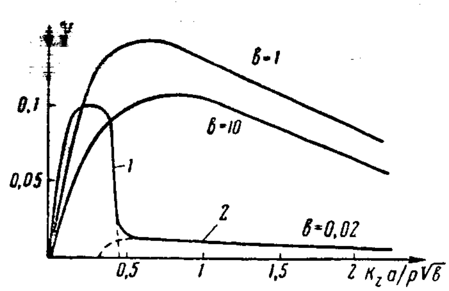

Fig. 1

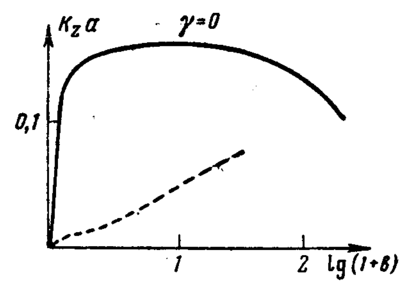

Fig. 2

The dimensionless quantity $k_z a/p\sqrt{b}$ is in abscissa axis and the dimensionless increment $\gamma = \mathrm{Im}\,\omega\sqrt{b}/k_y v_0$ – in ordinates.

The increment γ, conditioned by the build-up by resonance electrons, i.e. appearing at the expense of deduction, decreases proportionally to b^2 for small b, which is seen for example from (5). However, for very small b ($b \ll 0.1$), there appears an instability of hydrodynamic character in perturbations with a very great wave-length along z ($\alpha p \gg 1$). One may utilize for such perturbations the asymptotic expansion $Y(pz) \cong \frac{1}{pz} + \frac{1}{2p^3z^3}$, and we shall then obtain from (4)

$$\frac{z-\alpha}{z+\alpha} = 2p^2bz^2. \qquad (6)$$

It is fairly easy to obtain in that equation the condition of complex radical appearance (third degree equation). The condition has the form $\alpha^2p^2b > 4(1+\sqrt{5})^{-1} \times (3+\sqrt{5})^{-2} \approx 0{,}09.$ on the curve $b = 0.02$ (Fig. 1), parcel 1 corresponds to that instability.

Plotted is in Fig. 2 the instability boundary $k_za = f(b)$ for $p = 1/43$,* but for $b \gg 1$ the boundary value of k_za is determined by ionic damping and results nearly independent from b. The dashed line of Fig. 2 shows, that for fixed values of b the increment γ reaches the maximum along k_za.

One may carry out with the aid of the dispersion equation (3) the investigation of instability in a more generalized case of non-isothermic non-isotropic plasma, but this problem is beyond the framework of the present paper. Let us only note that inasmuch as the wave build-up considered here is due to electrons, the increment Im ω is basically determined by electron temperature. In particular, instability also takes place in the boundary case of cold ions ($T_i \ll T_e$). Hence it follows in particular, that alongside with the effect of Larmor radius' finiteness, the ion inertia leads also to the shift of wave's phase, contributing to instability.

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* For small b it is determined by the condition Re $z < \alpha$ and approximately $k_za = b$.

4. Thus, we have shown that a nonuniform isothermic plasma in a strong magnetic field is unsteady relative to the build-up of drift-type waves with a transverse wavelength of the order of the mean Larmor radius of ions ρ_i. The corresponding perturbations are strongly stretched along the magnetic field's lines of force, and that is why the instability, realistically investigated here, may only manifest itself in the configurations, whose longitudinal length L exceeds by one order the transverse length a.

Since the transverse length of the wave of growing perturbations has at small values a/L the order of magnitude ρ_i, while the development time of oscillations is determined by the quantity $a/\rho_i v_i$, where $v_i = \sqrt{T/m_i}$, one may think that the pulsations developing on account of instability lead to plasma diffusion across the magnetic field with a diffusion coefficient of the order of $\rho_i^2 v_i/a$. This coefficient drops as H^{-2} with the growth of the magnetic field, but in its absolute magnitude it may significantly exceed the diffusion coefficient on account of collisions.

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*** THE END ***

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